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A REVIEW OF "AT REST" DROPLET GROWTH EQUATIONS FOR  
CONDENSING NITROGEN IN TRANSONIC CRYOGENIC WIND TUNNELS

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A REVIEW OF "AT REST" DROPLET GROWTH EQUATIONS  
FOR CONDENSING NITROGEN IN TRANSONIC CRYOGENIC WIND TUNNELS

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SUMMARY

Droplet growth equations are reviewed in the free-molecular, transition, and continuum flow regimes with the assumption that the droplets are "at rest" with respect to the vapor. As comparison calculations show, it is important to use a growth equation designed for the flow regime of interest. Otherwise, a serious over-prediction of droplet growth may result. The growth equation by Gyarmathy appears to be applicable throughout the flow regimes and involves no iteration. His expression also avoids the uncertainty associated with selecting a mass accommodation coefficient and consequently involves less uncertainty in specifying adjustable parameters than many of the other growth equations.

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## INTRODUCTION

When condensation is encountered in a wind tunnel, the test gas may cease to properly simulate flow in ambient-temperature air. Understanding the condensation process is essential for maximum utilization of cryogenic wind tunnels because the onset of condensation effects determines the minimum useful operating temperature which, in turn, determines the maximum benefits of cryogenic operation (see references 1 to 4). An experimental program at the NASA Langley Research Center has utilized the 0.3-meter transonic cryogenic tunnel to establish a data base for determining the onset of condensation effects under a variety of test conditions. Proper analysis of the data from the 0.3-m tunnel and extension of the results to larger cryogenic tunnels such as the National Transonic Facility (see references 5 and 6) require the use of equations for droplet formation and growth. With the assumption of no relative motion between the droplet and its vapor, this report reviews and evaluates droplet growth equations as they apply to flow in transonic cryogenic wind tunnels using nitrogen as the test gas.

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# SYMBOLS

$c_p$	heat capacity of liquid, J/(kg.K)
$D$	molecular impingement rate per unit area, $1/(m^2 \cdot sec)$
$D_e$	molecular evaporation rate per unit area, $1/(m^2 \cdot sec)$
$k$	Boltzmann's constant, J/K
$Kn$	Knudsen number, defined in equation (2)
$\lambda$	mean free path, m
$L$	latent heat, J/kg
$m_v$	molecular mass of condensing vapor
$P$	pressure, $N/m^2$
$Pr$	Prandtl number
$r$	distance from the center of a droplet, m
$r_c$	critical radius of droplet, defined in equation (1), m
$R_v$	specific gas constant of vapor, J/(kg.K)
$t$	time, sec
$T$	temperature, K
$U$	internal energy, J/kg
$\omega$	velocity, m/sec
$x$	position, m
$\alpha$	mass accommodation coefficient
$\gamma$	ratio of specific heats
$\eta$	gas viscosity, kg/(m.sec)
$\lambda$	thermal conductivity, J/(m.K.sec)
$\xi$	thermal accommodation coefficient
$\rho$	density, $kg/m^3$
$\sigma$	surface tension, N/m

## Subscripts

d	relating to liquid droplet
s	relating to saturated value
v	relating to vapor

## GROWTH ENVIRONMENT

Before the droplet growth equations themselves are presented, it is useful to review the criterion for droplet growth, the impact of the flow regime and the composition of the condensing medium on the growth equations, and the assumption of no relative motion between the droplet and its vapor.

### Criterion for Growth

For a droplet surrounded by its vapor, growth is characterized by the vapor molecules condensing on its surface. Continued growth will be ensured when the radius of the droplet becomes greater than a critical radius,  $r_c$ , defined in reference 7 as

$$r_c = \frac{2\sigma}{R_v T_v \rho_d \ln(p_v/p_s)} \quad (1)$$

where  $\sigma$  is the surface tension,  $R_v$  is the specific gas constant,  $T_v$  and  $p_v$  are the temperature and pressure of the vapor,  $\rho_d$  is the density of the droplet,  $p_s$  is the saturation pressure at the vapor temperature, and  $p_v/p_s$  is the supersaturation ratio. A droplet with radius  $r_d = r_c$  will be in unstable equilibrium with its vapor while a droplet of radius  $r_d < r_c$  will evaporate. The growth of a droplet with  $r_d > r_c$  is dependent on the size of the droplet relative to the local mean free path and on the composition of the surrounding medium. These two factors will be examined individually.

### Flow Regime

The first factor governing droplet growth is the size of the droplet. The Knudsen number,  $Kn$ , is a parameter used to define different regimes of droplet growth, and is given as

$$Kn = \frac{\ell}{2r_d} \quad (2)$$

where  $\ell$  is the mean free path and  $2r_d$  is the droplet diameter. A growing droplet formed by the condensing vapor will initially be smaller than the mean free path of the vapor, and therefore  $Kn \gg 1$ . This defines the free-molecular regime. The growth of the droplet is determined by molecular processes at the surface of the droplet, and kinetic theory provides the governing equations in this case. As the droplet grows, it will eventually become larger than the mean free path of the surrounding vapor and then can be considered to be in a continuum flow regime, where  $Kn \ll 1$ . Macroscopic transport phenomena will describe the further growth of the droplet and molecular processes will no longer be necessary for the analysis. The situation where the droplet diameter is approaching and then surpassing the mean free path,  $Kn \approx 1$ , is called the transition regime. Here the importance of a molecular description lessens while a macroscopic description becomes more important.

For many condensation problems, it can be assumed that the droplet is entirely confined to the free-molecular flow regime during its lifetime, thus avoiding the transition and continuum regimes. However, the cryogenic nitrogen-gas transonic wind tunnels offer a different environment than normally encountered for droplet growth. The two primary differences are the higher densities and longer dwell-times involved. In the 0.3-meter tunnel for example, where typical stagnation conditions might be  $T = 120$  K and  $P = 5 \times 10^5$  N/m<sup>2</sup> for a test section Mach number of 0.8, the mean free path for nitrogen gas is on the order of  $10^{-8}$  m in the test section. For many of the past condensation investigations in supersonic and hypersonic tests, the mean free path has been on the order of  $10^{-7}$  m (see references 8 and 9). This difference in mean free path implies that the droplets, by



equation (2), physically do not have to be as large to attain a given value of  $Kn$ . Also, since the molecular mean free time between collisions is also proportional to mean free path, the smaller mean free path will lead to higher collision rates and provide a higher growth rate. Furthermore, whereas times for growth in supersonic and hypersonic tunnels are often on the order of  $10^{-4}$  sec (see, for example, reference 8), the times for growth in the test section of the 0.3-m transonic tunnel can be as long as  $10^{-2}$  sec. When the longer dwell-times are coupled with the smaller mean free path, the assumption that a free-molecular analysis is sufficient is no longer valid. Thus what is needed for transonic cryogenic tunnels is a growth equation or equations that describe droplet growth in free-molecular, transition, and continuum regimes.

#### Composition of Condensing Medium

While the flow regime is important for droplet growth, the composition of the medium surrounding the droplet is also important. The 0.3-meter tunnel uses nitrogen as the test gas and is consequently a single-component system. A more common situation described in the literature is the two component system which has the condensing vapor dispersed in an inert carrier gas, where "vapor" is used to refer to the condensable component and "gas" refers to the noncondensable component.

In free-molecular flow, condensation is determined by the kinetics of molecular collisions. Once a droplet starts growing, it can only continue to grow if the latent heat released by the condensing vapor molecules is taken away. Heat can be removed from the droplet in two ways - by evaporation or by collisions with either vapor or gas molecules where the rebounding molecules receive heat energy from the droplet. The presence of an inert carrier gas can increase the probability that a vapor molecule will condense on the droplet because the inert molecules can be continually removing the latent heat away from the droplet.

In continuum flow, growth depends on the transport processes of mass diffusion and heat conduction. With an inert carrier gas present, heat is again carried away by the gas so that vapor-droplet collisions are likely to result in condensation. On the other hand, the limiting factor for growth with an inert gas present becomes the diffusion of the condensable vapor through the gas - the droplet can grow only as fast as the vapor molecules can diffuse to it. With no carrier gas present, diffusion is not important but the heat generated by condensation must be carried away before the droplet can grow. Therefore, with pure nitrogen gas, heat conduction is the limiting factor for droplet growth in the continuum regime as well as in the free-molecular regime.

#### Droplet Relative Motion

An additional assumption used to simplify the problem of droplet growth concerns the relative motion of the droplets with respect to the

vapor. It is normally assumed that the droplets are "at rest" relative to the surrounding medium so that convective heat transfer effects can be neglected. Consequently, it is necessary to examine under what conditions this "at-rest" assumption will be valid. The breakdown will occur when the forces entraining the droplet in the flow are not strong enough to keep its velocity the same as the velocity of the vapor. For small droplets with  $Kn \gg 1$ , the molecular forces are normally sufficient to keep the droplet at the vapor velocity. For a continuum environment with  $Kn \ll 1$ , the droplet will stay with the vapor as long as the viscous forces tending to keep the velocities the same are as strong as the inertial forces on the droplet. The larger the velocity gradient encountered, then the smaller the droplet has to be to stay with the vapor, as can be seen by equating the inertial force term to the viscous drag term as determined by Stokes equation. Following the reasoning of Wegener and Mach in reference 7, one arrives at the following expression for the maximum allowable velocity gradient for negligible velocity difference:

$$\frac{d\omega_d}{dx} = \frac{9}{2} \frac{\eta}{\rho_d} \frac{1}{r_d^2} \left( \frac{\omega}{\omega_d} - 1 \right) \quad (3)$$

where  $\omega_d$  is the velocity of the droplet,  $\omega$  is the velocity of the gas,  $\eta$  is the gas viscosity,  $\rho_d$  is the liquid density of the droplet, and  $r_d$  is the droplet radius. If it is assumed that the velocity of the droplet should not differ from  $\omega$  by more than 1%, the following table for maximum velocity gradients for nitrogen flow at 100 K can be constructed:

$r_d(m)$	$d\omega/dx \text{ (sec}^{-1}\text{)}$
$10^{-5}$	$4 \times 10^0$
$10^{-6}$	$4 \times 10^2$
$10^{-7}$	$4 \times 10^4$
$10^{-8}$	$4 \times 10^6$

Assuming Stokes flow in constructing the above table is a conservative assumption because as the Reynolds number of the droplet approaches and then surpasses 1, the actual aerodynamic drag on the sphere will become larger than that predicted by Stokes flow (see reference 10) and will consequently allow the droplet to experience larger gradients than tabulated above.

To summarize, for transonic cryogenic wind tunnels using nitrogen as the test gas, there is no inert gas present and heat conduction away from the droplet will be the limiting factor for the rate of droplet growth. Because of the high densities involved and the long dwell-times, droplet growth in the continuum regime, as well as the free-molecular regime, must be considered. The equations for droplet growth, as presented below, will only be valid if the velocity gradients are less than those given in the above table for the droplet size of interest.

#### EQUATIONS

Growth equations applicable to the environment of interest in cryogenic wind tunnels are outlined along with the fundamental assumptions made by

the respective investigators. The equations are presented according to their flow regime of applicability.

#### Free-Molecular Regime

Droplets formed by the condensing vapor (homogeneous nucleation) normally begin growing in the free-molecular regime, where rarefied gas assumptions hold. In particular, kinetic theory (see reference 9) provides an expression for the mass that crosses unit area per second toward one side. This molecular impingement rate per unit area,  $D$ , on a droplet is given by

$$D = \frac{P_v}{(2\pi m_v k T_v)^{1/2}} \quad (4)$$

where the values of the variables are taken from the condensing vapor and  $m_v$  is the molecular mass and  $k$  is Boltzmann's constant. In order to determine the flow rate leaving the droplet, it is assumed that at critical radius,  $r_c$ , the droplet is in quasi-equilibrium and evaporates at the same rate as impingement occurs. Then the evaporation rate per unit area,  $D_e$ , can be taken to be equal to the impingement rate on the droplet, or

$$D_e = \frac{P_{r_d}(T_d)}{(2\pi m_v k T_d)^{1/2}} \quad (5)$$

where  $T_d$  is the droplet temperature and  $P_{r_d}(T_d)$  is the value of pressure which would just make the droplet of radius  $r_d$  a critical-sized droplet. Thus, the difference between the condensing, incoming mass and the evaporating, outgoing mass is used to calculate the growth rate. In order to complete the growth rate equation, a mass accommodation coefficient,  $\alpha$ , is also needed. The value of  $\alpha$  indicates the fraction of molecules that actually penetrate into the droplet when they strike it. Altogether, for the free-molecular regime the growth rate is given by

$$\frac{dr_d}{dt} = \alpha \frac{m_v}{\rho_d} \left[ \frac{P_v}{(2\pi m_v k T_v)^{1/2}} - \frac{P_{r_d}(T_d)}{(2\pi m_v k T_d)^{1/2}} \right] \quad (6)$$

A disadvantage with this expression is that the droplet temperature,  $T_d$ , is an unknown quantity.

Hill, in reference 11, extends the above results by providing an equation to solve for the droplet temperature,  $T_d$ . He does this by balancing the net rate of energy flux to the droplet with the rate of increase of its internal energy. The energy equation he obtains, after being simplified for a single component gas, is

$$\begin{aligned} K_v \alpha \left( 1 - \frac{P_{r_d}(T_d)}{P_v} \sqrt{\frac{T_d}{T_v}} \right) - K_v (1 - \alpha) \xi \left( \frac{T_d}{T_v} - 1 \right) \\ + \alpha \left( 1 - \frac{P_{r_d}(T_d)}{P_v} \sqrt{\frac{T_v}{T_d}} \right) (A - 1) \frac{\gamma}{\gamma - 1} = 0 \end{aligned} \quad (7)$$

where

$$K_v = \frac{1}{2} \frac{\gamma + 1}{\gamma - 1},$$

$\alpha$  is the mass accommodation coefficient,  $\xi$  is the thermal accommodation coefficient, and

$$A = \frac{(\gamma - 1)L}{\gamma R T_v},$$

where  $L$  is the latent heat of the condensing vapor. The first term in equation (7) shows the energy difference between molecules condensing on the droplet and molecules evaporating from the surface of the droplet. The second term is the energy of reflected vapor molecules from the surface of the droplet. The last term is the increase of the internal energy of the droplet neglecting the energy needed to raise the temperature of the droplet, which is shown to be negligible for the small droplets expected in free-molecular flow. In order to solve for  $r_d$ ,  $T_d$ , and  $P_d$ , an iteration procedure is required.

In reference 9, Wu has developed a set of equations that bypasses the iteration necessary for Hill's expression. Wu starts with kinetic theory and defines a new mass accommodation coefficient,  $\alpha'$ , as

$$\alpha' = \alpha \left\{ 1 - \frac{P_{r_d}(T_d)}{P_v} \sqrt{\frac{T_v}{T_d}} \right\} \quad (8)$$

and calls it the "effective" mass accommodation coefficient. The last term is the ratio of the evaporation rate over the condensation rate. The growth equation can then be written as

$$\frac{dr_d}{dt} = \alpha' \frac{m_v}{\rho_d} \frac{P_v}{(2\pi m_v k T_v)^{1/2}} \quad (9)$$

Next, Wu specifies the conditions of the droplet and its environment so that simplifying assumptions can be made. With these assumptions Wu modifies  $\alpha'$  by developing four separate equations based on different assumptions. Referring to figure 1, the assumed conditions of the droplet include thermal and near-thermal equilibrium in addition to saturation and near-saturation temperature. For the present case of no inert carrier gas, Wu's approximation of the droplet being close to saturation temperature is satisfactory. With this assumption, Wu finds that  $\alpha'$  becomes

$$\alpha' = \frac{\alpha \left\{ 1 - \sqrt{\frac{T_v}{T_s}} \left[ 1 - \frac{Lm_v}{kT_s} \left( 1 - \frac{T_v}{T_s} \right) \right] \right\}}{1 + \frac{\alpha}{K_v} \sqrt{\frac{T_v}{T_s}} \frac{Lm_v}{kT_s} \left( \frac{Lm_v}{kT_v} - \frac{1}{2} \right)} \quad (10)$$

where  $T_s$  is the saturation temperature for a droplet of radius  $r_d$  at a



vapor pressure of  $P_v$ . (The curve labeled  $P_{r_d}$  in figure 1 describes pressures at which a droplet of radius  $r_d$  is just critical-sized. That is, for a given value of  $T_d$ , if  $P_v > P_{r_d}$  then the droplet will grow. If  $P_v < P_{r_d}$ , then the droplet will evaporate.)

In another method, Oswatitsch in reference 12 also used kinetic theory but in a different way. Oswatitsch took the expression for heat transfer and matched it to the expression for mass increase times the heat of vaporization. This resulted in the following equation for growth rate:

$$\frac{dr_d}{dt} = \frac{3}{8} \frac{P_v}{\rho_d L} \sqrt{3 \frac{R}{T_v}} (T_d - T_v) \quad (11)$$

This equation was specifically derived for a droplet condensing in its vapor without a carrier gas. Oswatitsch also derived an expression for droplet temperature by including the Clausius-Clapeyron vapor-pressure curve along with the heat transfer equations. This combination resulted in the following equation,

$$\frac{T_s - T_v}{T_d - T_v} = 1 + \frac{3}{2} \frac{1}{\alpha} \left( \frac{R T_s}{L} \right)^2 \quad (12)$$

which may be readily solved for  $T_d$ . The value of  $T_s$  used by Oswatitsch in equation (12) is just the vapor-pressure curve temperature,  $T_s (r_d = \infty, P_v)$ , and does not take into consideration the droplet radius. Consequently,  $T_d$  and  $\frac{dr_d}{dt}$  are both independent of droplet size.

#### Continuum Regions

In reference 12, Oswatitsch has also developed an equation for continuum flow in which droplet growth is determined by thermal conduction on a macroscopic scale. The final expression is

$$r_d^2 = \frac{2 \lambda_v (T_d - T_v)}{L \rho_d} t \quad (13)$$

or

$$\frac{dr_d}{dt} = \frac{\lambda_v (T_d - T_v)}{L \rho_d r_d} \quad (14)$$

where  $\lambda_v$  is the thermal conductivity of the vapor. Unfortunately, Oswatitsch does not give an expression for  $T_d$  for this case as he did for the free-molecular case. Consequently,  $T_d$  will be assumed in the present paper to be equal to  $T_s (r_d, P_v)$ . Oswatitsch proposed that his first expression,

equation (11), be used while the droplet is in free-molecular flow while the second equation (13), be used for continuum flow, but there is uncertainty in describing droplet growth in the transition regime.

Fuchs in reference 13 does a survey of evaporation (or condensation in reverse) of droplets in the presence of an inert carrier gas. Although the inert carrier gas invalidates many of his equations for the present single component system, the beginning of Fuchs' derivation starts with an equivalent to equation (14) before he begins to make assumptions appropriate for the presence of an inert carrier gas.

#### All Flow Regimes

Gyarmathy in reference 14 uses diffusion and heat conduction to develop a continuum equation which closely approximates droplet growth in the free-molecular regime as well. For a pure vapor Gyarmathy's droplet growth equation reduces to

$$\frac{dr_d}{dt} = \frac{\lambda_v}{\rho_d L} \frac{1 - \frac{r_c}{r_d}}{1 + \left( \frac{2 \sqrt{8\pi}}{1.5 Pr_g \xi} \frac{\gamma}{\gamma + 1} \right) Kn} \cdot \frac{(T_s - T_v)}{r_d} \quad (15)$$

where  $Pr_g$  is the Prandtl number for the gas,  $T_s$  is the corresponding value of temperature for a value of  $P_v$  on the vapor-pressure curve, and  $\gamma$  is the ratio of specific heats. The fact that the applicability of this equation extends from the continuum regime to the free-molecular

regime makes it the most useful of the equations reviewed. Also, equation (15) does not depend on the mass accommodation coefficient. However, Gyarmathy's expression neglects (as do Oswatitsch's continuum equation and all of the free-molecular equations) the change in internal energy of the droplet due to a varying droplet temperature

$$\Delta U_d = \frac{4\pi r_d^3}{3} \rho_d C_p \Delta T_d , \quad (16)$$

where  $C_p$  is the heat capacity of the liquid.

While Gyarmathy states that this assumption is excellent for droplets of radius  $10^{-6}$  m or less, if droplets much larger than  $10^{-6}$  m pass through significant temperature gradients in the flow, then the droplet growth equation should be expanded to include this term.

## RESULTS AND DISCUSSION

The droplet growth rates predicted by the various growth rate equations are presented for a pure nitrogen gas with  $P_v = 2 \times 10^5 \text{ N/m}^2$  and  $T_v = 73 \text{ K}$ . The different rates are compared to the rate predicted by Gyarmathy, equation (15), because equation (15) is designed to be applicable in all of the flow regimes. Shaded areas representing uncertainty in the equations are also shown to indicate the degree of uncertainty that exist in specifying parameters used in the various equations.

The comparisons between the free-molecular expressions of Hill, Wu, and Oswatitsch and the equation of Gyarmathy are shown respectively in figures 2, 3, and 4. The uncertainty in what value to assign to mass accommodation coefficient,  $\alpha$ , is responsible for the large uncertainty bands in the three graphs.

It is not clear what the value of  $\alpha$  should be or whether it should be the same for one-component or two-component flows. For the case of a single component test-gas, steam, Hill found that experimental nozzle flow data correlated best with his growth equation if  $\alpha$  was taken to be 0.04. Similarly, Oswatitsch stated that his own data with steam suggested a value for  $\alpha$  of 0.03 in order to match the data to the predictions of his equations. On the other hand, Wegener and Wu in reference 15 recommend a value of  $\alpha$  of 0.7 for growth equation developed by Wu. The presence of an inert gas in their data may be responsible for some of the difference in  $\alpha$ .

In recent years, determination of mass accommodation coefficients from film condensation experiments have led to estimations of  $\alpha$  much higher than 0.04. For example, Mills and Seban in reference 16 have

suggested that the value of  $\alpha$  for steam should be somewhere between 0.45 and unity. This range of values, if also applicable for droplet condensation, would seem to support the value recommended by Wegener and Wu but would raise some questions about the experimental comparison of Oswatitsch and of Hill in addition to questioning the accuracy of equation (15) for both the free-molecular and transition regimes.

Consequently, in constructing the uncertainty bands for the free-molecular calculations shown in figures 2, 3, and 4, a heavy line is drawn for the value of  $\alpha$  used by the respective author while the uncertainty band extends over a range of  $\alpha$  from 0.03 to 0.7. While the choices of  $\alpha$  vary between Hill, Wu, and Oswatitsch, the uncertainty band regions are very similar between the three authors. Also, at small values of  $r$ , equation (15) falls inside the uncertainty regions with the exception of the one calculated from Oswatitsch, which does not depend on the radius of the droplet and consequently predicts a constant value of  $\frac{dr}{dt}$ . This is in error at the smaller values of  $r_d$  approaching  $r_c$ , where  $T_d = T_v$  and where the droplet should have a zero growth rate.

In order to present a comparable uncertainty band in the continuum expression of Oswatitsch, an examination of the probable derivation of equation (14) was undertaken to see if any possible difficulty existed in specifying the continuum parameters as occurred for  $\alpha$  in the free-molecular expressions. As shown in the appendix, a problem does appear in having to assume that  $\lambda_v$  is independent of temperature in order to arrive at equation (14). However, in most cases  $\lambda_v$  is nearly

porportional to temperature, so if one assumes that

$$\lambda_v(T) = \lambda_v(T_d) \frac{T}{T_d} \quad (17)$$

then the following growth equation can be derived (see the appendix)

$$\frac{dr_d}{dt} = \frac{\lambda_v(T_d)(T_d^2 - T_v^2)}{2L \rho_d r_d T_d} \quad (18)$$

Nevertheless, as seen in figure 5 where the heavy line represents the original equation of Oswatitsch and the uncertainty band extends to the line corresponding to equation (18), there is not a great deal of difference between equations (14) and (18) for the present  $P_v$  and  $T_v$  conditions. Values for the parameters in these equations were taken from, or extrapolated from, reference 17.

As seen in figure 5, equation (14) and equation (15) asymptotically approach one another as the continuum regime is reached. This agreement is to be expected as the two equations are equal in the limit of  $Kn \rightarrow 0$  and  $\frac{r_c}{r_d} \rightarrow 0$ .

A similar attempt was made to determine what assumptions had been made in deriving equation (15). Unfortunately, reference 14

cited for Gyarmathy does not contain the necessary information to retrace a derivation, so his assumptions could not be examined. In particular, it is not clear how the numerical factor  $\frac{2\sqrt{8\pi}}{1.5}$  in the denominator of equation (15) is determined. Nevertheless, as a rough approximation to the possible uncertainty in equation (15), one may substitute into the equation a range of values for the parameters  $Pr_g$  and  $\gamma$  ( $\xi$ , the thermal accommodation coefficient, is consistently recognized to be unity in the literature so no change was made to this parameter). If the derivation of equation (15) assumed an ideal gas, then perhaps  $\gamma$  should be taken to be 1.4 instead of the real-gas value of 1.53. Similarly, perhaps  $Pr_g$  should be taken to be the ambient temperature value of 0.726 instead of the real-gas value of 0.844. The range in values that results from the various combinations of the above values of  $\gamma$  and  $Pr_g$  are shown in figure 6, where the heavy line represents equation (15) with the real-gas values of  $\gamma$  and  $Pr_g$ .

As shown in figures 2 to 6, care must be taken in selecting an equation to predict droplet growth rate. Serious over-prediction of droplet growth can result if free-molecular expressions are applied to either the transition or continuum regimes or if continuum equations are applied to the transition or free-molecular regimes. In the free-molecular equations, setting the value of  $\alpha$ , the mass accommodation coefficient, is a problem since a range of  $\alpha$  between 0.03 and 0.7 can result in a factor of 3 difference in growth rate. Equation (15) seems to yield reasonable magnitudes of growth rate in the free-molecular limit as well as in the continuum limit and appears to be well-behaved in the transition regime. Also,



equation (15) has less uncertainty in specifying parameters than the free-molecular expressions. Consequently, for the present study equation (15) is the preferred representation for droplet growth in all flow regimes.

## SUMMARY OF RESULTS

It is important to use a droplet growth equation that is designed for the flow regime of interest - free molecular, transition, or continuum. A serious over-prediction of droplet growth may result if an equation is incorrectly applied. The growth equation by Gyarmathy appears to be applicable throughout the flow regimes and therefore offers an easy solution for droplet growth. The expression by Gyarmathy also avoids the uncertainty associated with selecting a mass accommodation coefficient. Care, however, must be taken when droplets are larger in radius than  $10^{-6}$  m. First, possible differences in velocity between the gas and the liquid droplet can occur if pressure gradients are encountered because of the inertia of the relatively large droplets. Second, the growth rate equations have neglected the heat capacity of the droplets and this may lead to inaccuracies if the large droplets encounter temperature gradients.

## APPENDIX

### CONTINUUM GROWTH EQUATIONS AND THEIR THERMAL CONDUCTIVITY DEPENDENCE

Expressions for continuum droplet growth may be derived by equating the latent heat released during condensation to the heat conduction away from the spherical droplet. The exact form of the equation obtained depends, however, on whether or not the thermal conductivity,  $\lambda_v$ , is a function of temperature. If  $\lambda_v$  is assumed to be a constant, then the expression of Oswatitsch, equation (14), is readily obtained. If  $\lambda_v$  is not assumed constant, a modified version of equation (14) is obtained.

Starting with Fourier's law of one-dimensional, steady state, heat conduction, one may write

$$q = -\lambda_v A \frac{dT_v}{dr} \quad (A1)$$

where  $q$  is the heat flow in J/sec,  $A$  is surface area in  $m^2$ , and  $\frac{dT_v}{dr}$  is temperature gradient in K/m. For a spherically symmetric problem, equation (A1) becomes

$$q = -4\pi\lambda_v r^2 \frac{dT_v}{dr} \quad (A2)$$

Since  $q$  is just the total heat flux and is not a function of  $r$  or  $T_v$ , one may separate variables in equation (A2) and integrate. The integration, however, depends upon whether or not  $\lambda_v$  is a function of  $T_v$ , and, consequently,  $r$ .

#### Thermal Conductivity Constant

If  $\lambda_v$  is independent of  $T_v$  -- and consequently  $r$  -- the equation (A2) may be simply integrated from  $r = r_d$  to  $r = \infty$  and from  $T = T_d$  to  $T = T_v$ . The value of  $q$  is simply

$$q = 4\pi\lambda_v r_d (T_d - T_v) \quad (A3)$$

If the heat flow is equated to the latent heat multiplied by the rate of mass deposition on the droplet, one may write

$$q = 4\pi r_d^2 \frac{dr_d}{dt} \rho_d L \quad (A4)$$

Combining equations (A3) and (A4) gives

$$\frac{dr_d}{dt} = \frac{\lambda_v (T_d - T_v)}{L \rho_d r_d} \quad (14)$$

which is just the expression used by Oswatitsch.

#### Thermal Conductivity Variable

Separating variables and integrating equation (A2) leads to the following expression when  $\lambda_v$  is a function of temperature

$$q = -4\pi r_d \int_{T_d}^{T_v} \lambda_v(T) dT \quad (A5)$$

For many gases  $\lambda_v(T)$  can be approximated by

$$\lambda_v(T) = \lambda(T_d) \frac{T}{T_d} \quad (A6)$$

as was done by Penner in reference 18 (see page 282). With the above assumption, which appears very adequate for nitrogen, the right side of equation (A5) can be integrated to give

$$q = - \frac{2\pi r_d \lambda_v(T_d)}{T_d} \left\{ T_v^2 - T_d^2 \right\} \quad (A7)$$

Substituting for  $q$ ,

$$\frac{dr_d}{dt} = \frac{\lambda_v (T_d) \left\{ T_d^2 - T_v^2 \right\}}{2 L \rho_d r_d T_d} \quad (18)$$

which is just the corresponding droplet growth expression to equation (14)

assuming  $\lambda_v$  is a linear function of temperature.

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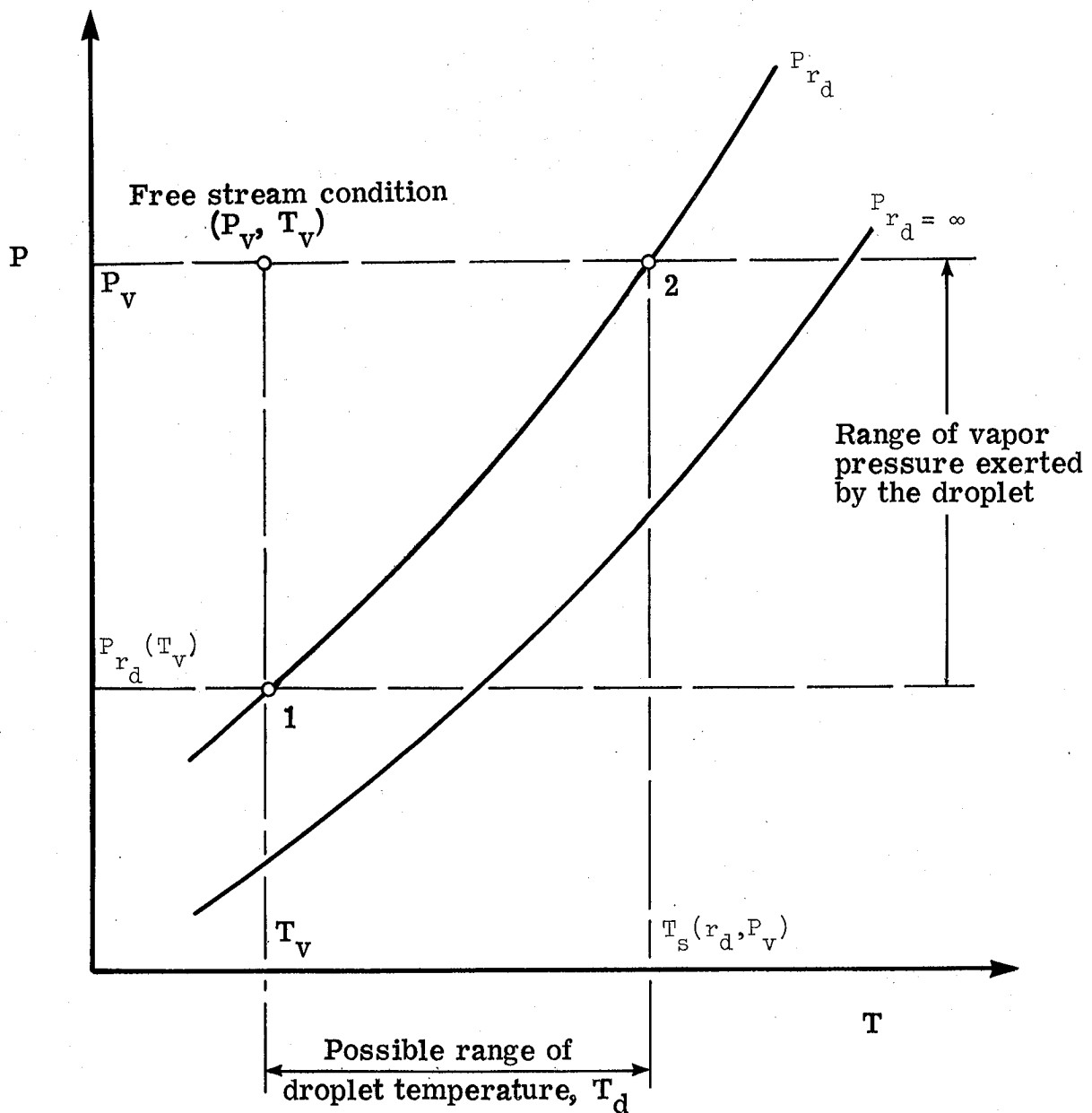


Figure 1.- Conditions of a growing droplet. Point 1 represents thermal equilibrium temperature assumption. Point 2 represents the saturation temperature assumption for droplet of radius  $r_d$ . (See reference 9.)

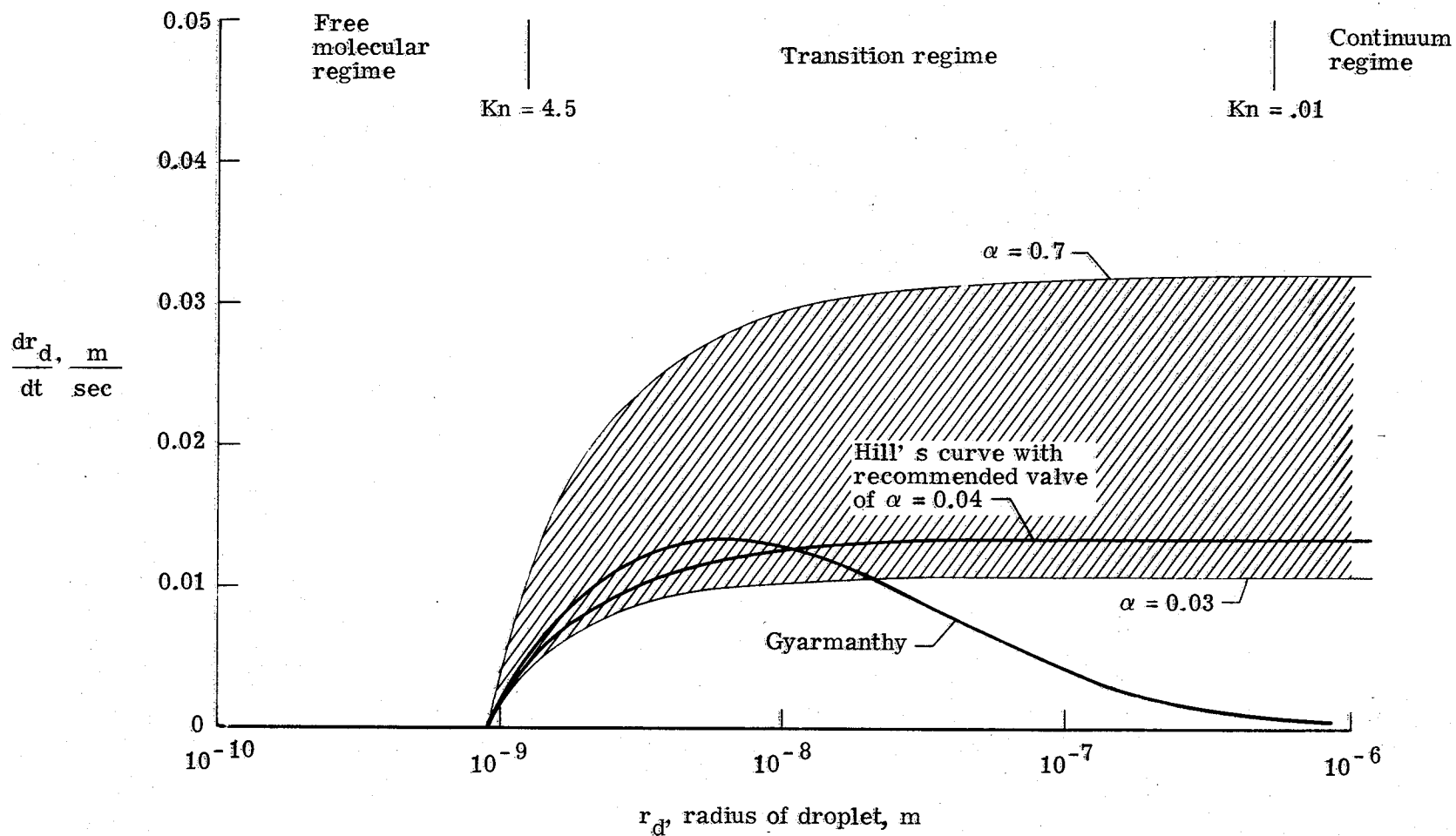


Figure 2.- Free-molecular growth rate of Hill calculated with different values of  $\alpha$  compared to growth rate predicted by Gyarmathy.

$$P_v = 2 \times 10^5 \text{ N/m}^2, T_v = 73 \text{ K, and } \ell = 1.12 \times 10^{-8} \text{ m.}$$

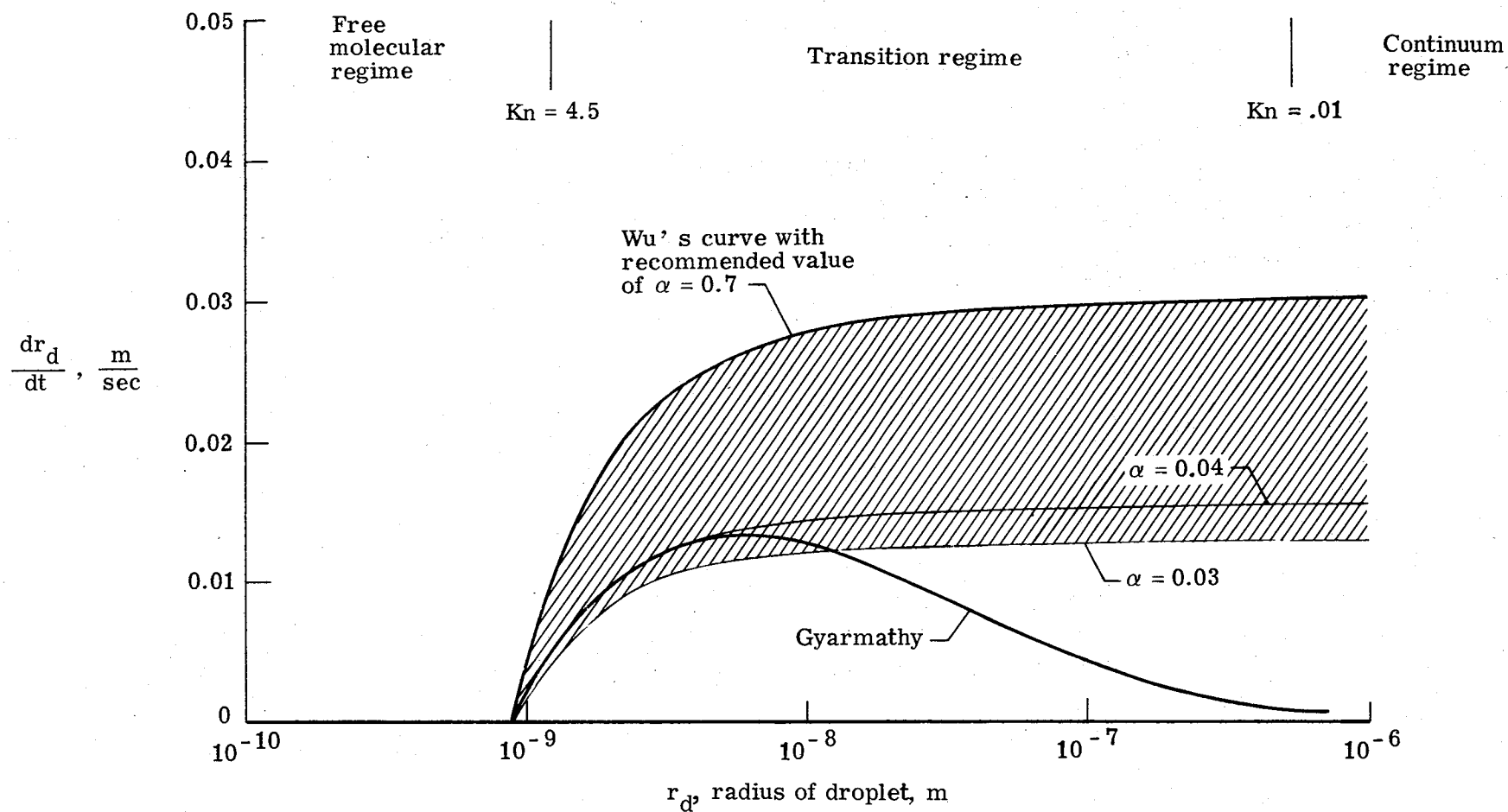


Figure 3.- Free-molecular growth rate of Wu calculated with different values of  $\alpha$  compared to growth rate predicted by Gyarmathy.

$$P_v = 2 \times 10^5 \text{ N/m}^2, T_v = 73 \text{ K, and } \ell = 1.12 \times 10^{-8} \text{ m.}$$

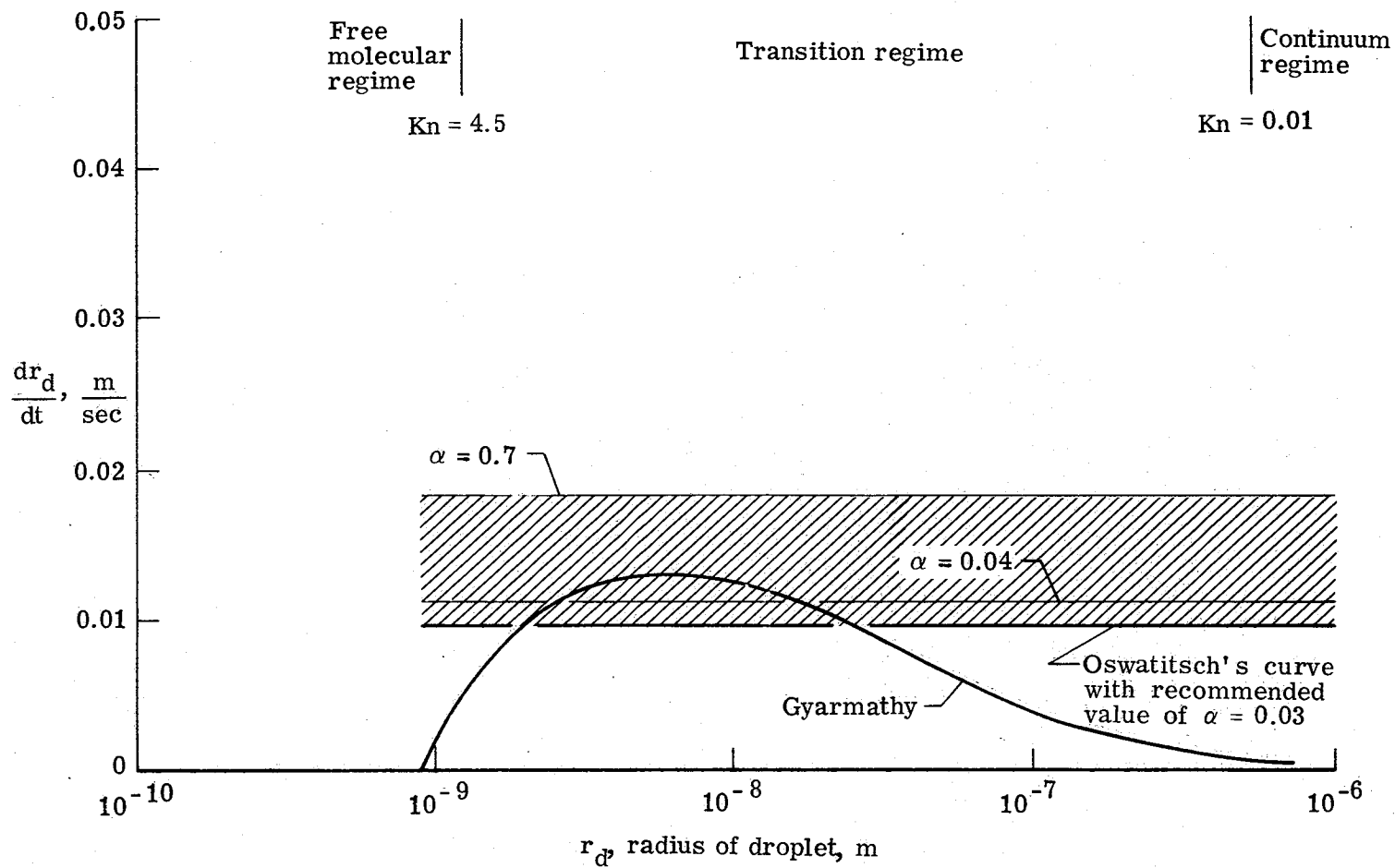


Figure 4.- Free-molecular growth rate of Oswatitsch calculated with different values of  $\alpha$  compared to growth rate predicted by Gyarmathy.

$$P_v = 2 \times 10^5 \text{ N/m}^2, T_v = 73 \text{ K, and } \ell = 1.12 \times 10^{-8} \text{ m.}$$

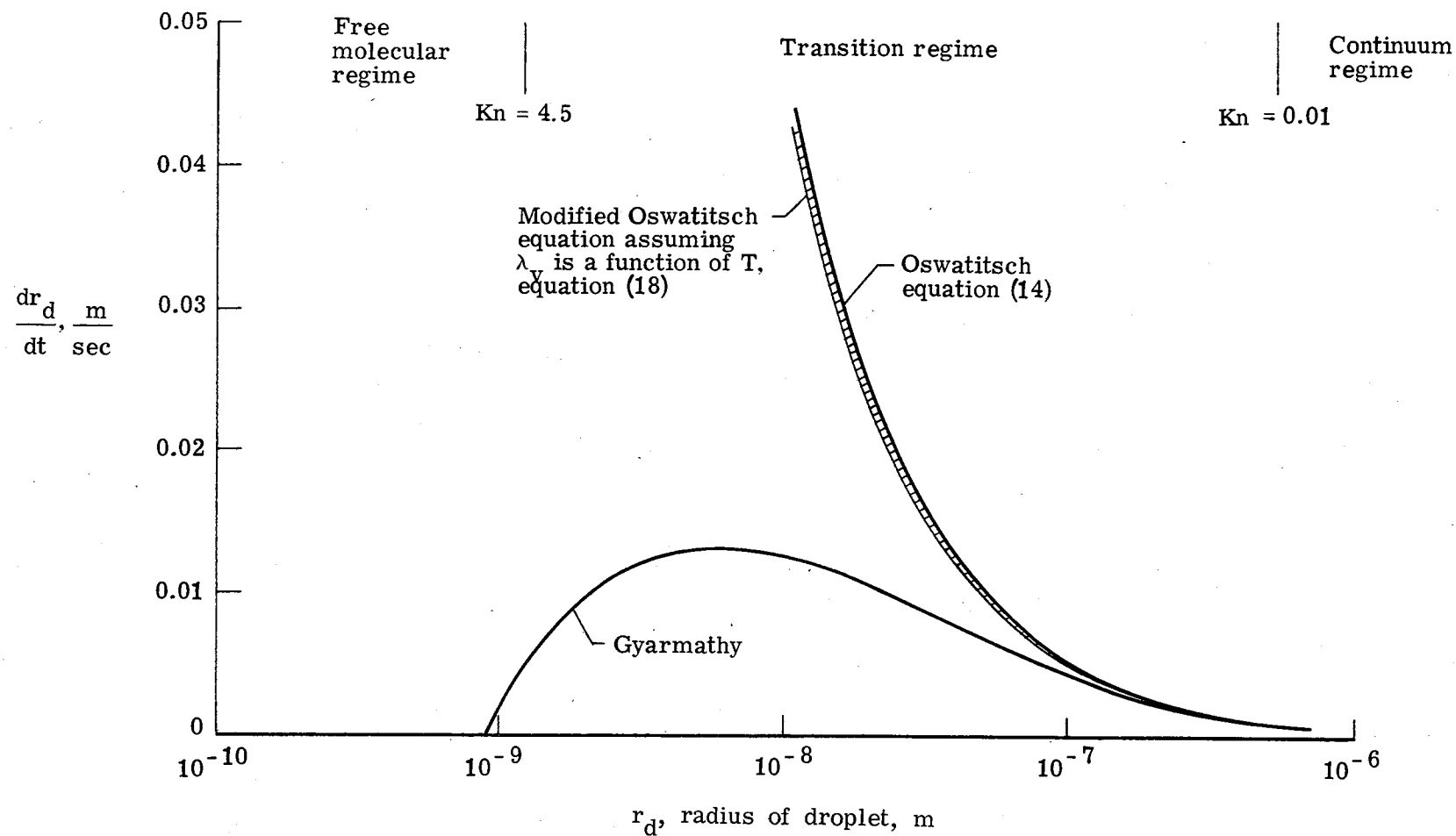


Figure 5.- Continuum growth rate of Oswatitsch assuming thermal conductivity,  $\lambda_v$ , is either a constant, equation (14), or a linear function of T, equation (18), compared to growth rate predicted by Gyarmathy.

$$P_v = 2 \times 10^5 \text{ N/m}^2, T_v = 73 \text{ K, and } \ell = 1.12 \times 10^{-8} \text{ m.}$$

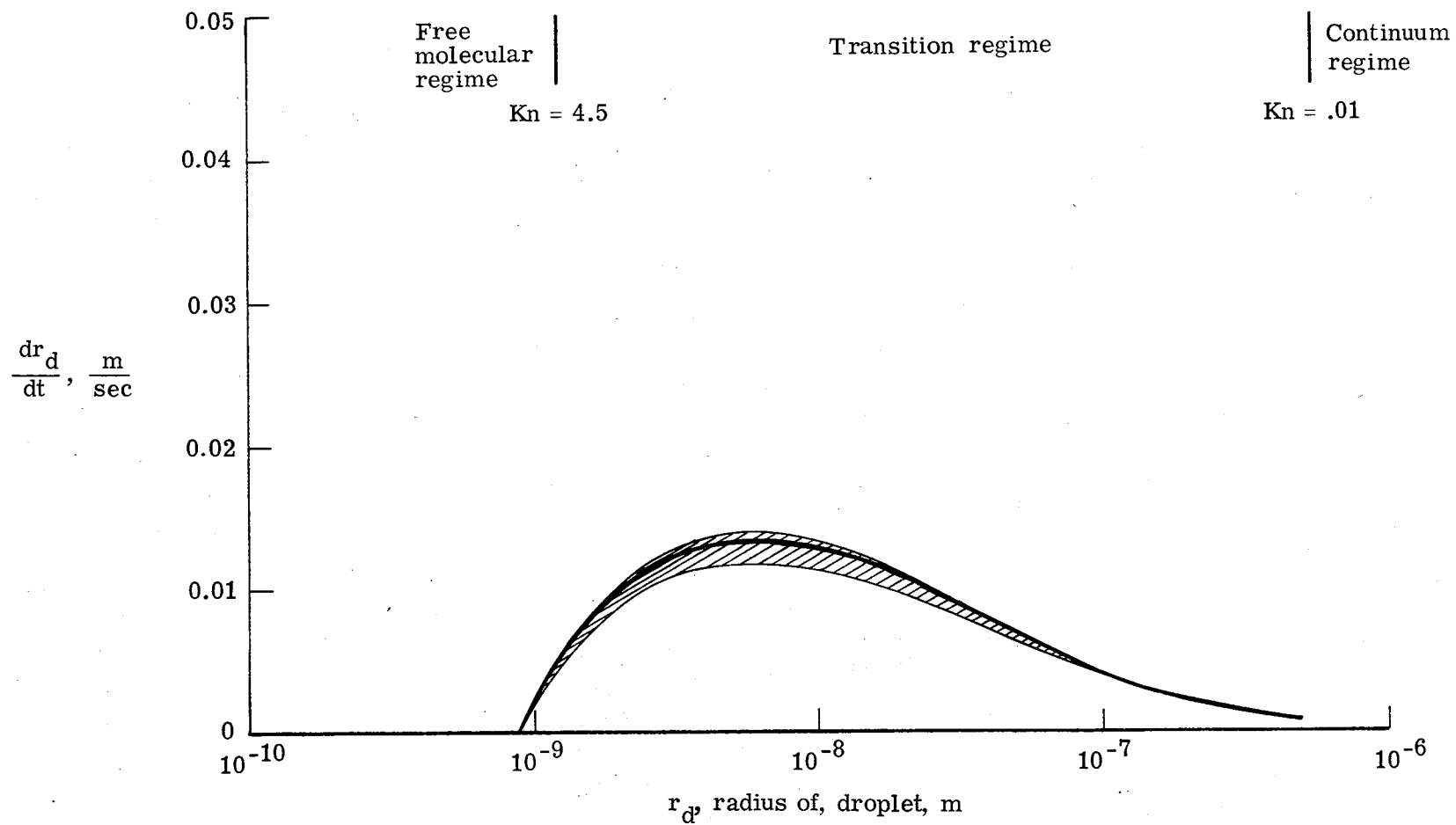


Figure 6.- Growth rate of Gyarmathy calculated with real-gas values of  $\gamma$  and  $Pr_g$  shown in heavy line. Ideal gas values of  $\gamma$  and  $Pr_g$  form band limits.<sup>g</sup>  
 $P_v = 2 \times 10^5 \text{ N/m}^2$ ,  $T_v = 73 \text{ K}$ , and  $\ell = 1.12 \times 10^{-8} \text{ m}$ .

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16. Abstract  Droplet growth equations are reviewed in the free-molecular, transition, and continuum flow regimes with the assumption that the droplets are "at rest" with respect to the vapor. As comparison calculations show, it is important to use a growth equation designed for the flow regime of interest. Otherwise, a serious over-prediction of droplet growth may result. The growth equation by Gyarmathy appears to be applicable throughout the flow regimes and involves no iteration. His expression also avoids the uncertainty associated with selecting a mass accommodation coefficient and, consequently, involves less uncertainty in specifying adjustable parameters than many of the other growth equations.					
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